Kinetic theory of Gases

Introduction:

↳ John Dalton discovered atom in 1803

↳ Kinetic theory in 1873

↓

Atoms / molecules constantly moves (MOTION)

Given by Maxwell & Boltzmann

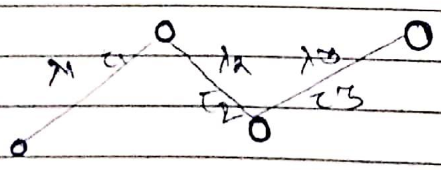
Assumption:

i) Intermolecular force is absent in ideal gases

* We study avg. properties of gases

Mean Free Path:

Avg. distance travelled by a molecule b/w 2 successive colln's.



$$\lambda_m = \frac{\lambda_1 + \lambda_2 + \dots + \lambda_n}{n}$$

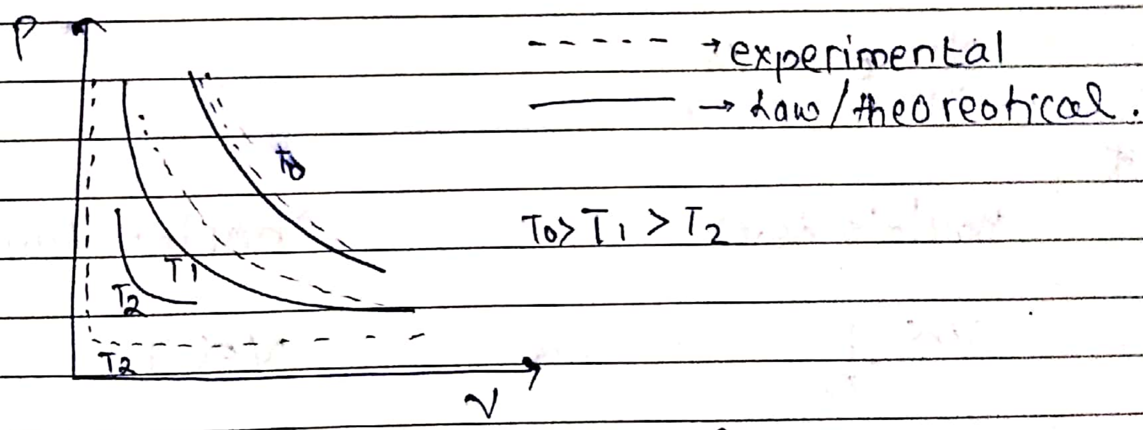
Avg relaxation time (τ)
Avg time period b/w 2 successive colln s.

$$\tau_m = \frac{\tau_1 + \tau_2 + \cdots \tau_n}{n}$$

IDEAL GAS Laws:

I] Boyle's law: (1661)
At, constant temp of gas, [isothermal]

$$V \propto \frac{1}{P}$$



High temp: (Real gas → ideal gas)
Gas law agrees experiment.

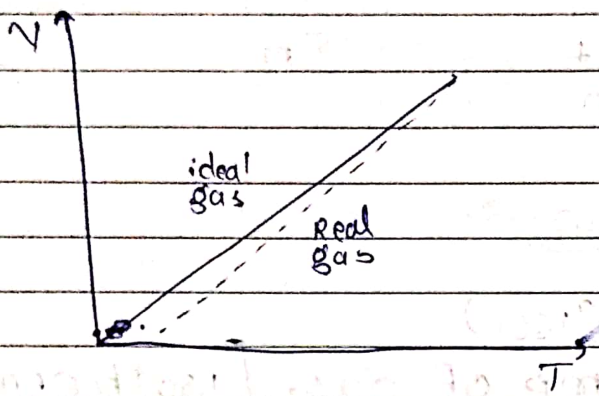
$$VP = \text{constant}$$

$$\boxed{P_1V_1 = P_2V_2} = \text{constant.}$$

II) 1783 : Charle's Law

At constant P

$V \propto T$



At high temp, law holds for all gases.

$$\frac{V}{T} = \text{constant}$$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

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*

Boyle's Law	Charles' Law
$V \propto \frac{1}{P}$	$V \propto T$
T = constant	P = constant

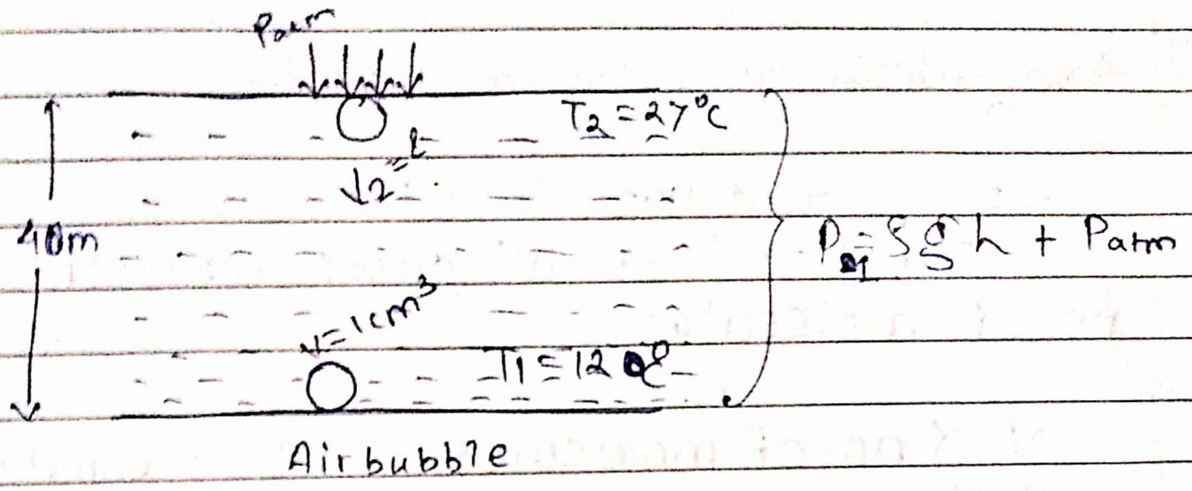
$$V \propto \frac{T}{P}$$

$$\frac{PV}{T} = \text{constant}$$

$$\boxed{\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}}$$

$P_{atm} = 1 atm$
$= 1.01 \times 10^5 \frac{N}{m^2}$

Q



~~$P_1V_1 = P_2V_2$~~

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\frac{(1.01 \times 10^5 + 10^3 \times 10 \times 40) \times 1cm^3}{285} = \frac{(1.01 \times 10^5) \times V_2}{300}$$

$$\frac{10^5 + 4 \times 10^5}{285} = \frac{10^5 \times V_2}{300}$$

$10^5(5)$

$$V_2 = \frac{300}{60}$$

$$V_2 = 5\,cm^3$$

III) Avogadro's hypothesis (1811)

At constant T & P :
Equal vol^m of all gases contains equal no. of molecules.

$V \propto$ no. of molecules (P & T → constant)
$V \propto$ no. of moles

$$\boxed{V \propto n}$$

Ideal Gas Equation:

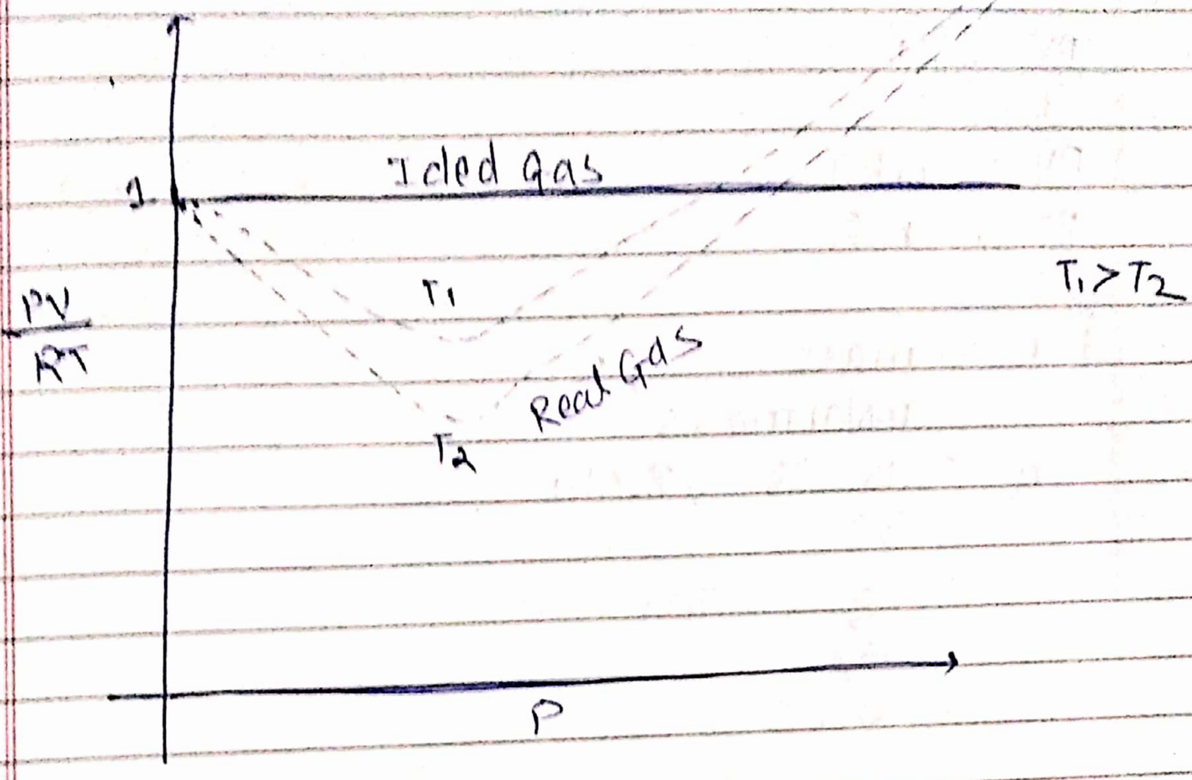
$V \propto \frac{1}{P}$ $\quad V \propto T$ $\quad V \propto n$

$$V \propto \frac{nT}{P}$$

$$V = \frac{RnT}{P}$$

$$\boxed{PV = nRT}$$

Where,
P = Pressure
V = Vol^m
R = Gas constant : 8.314 J/mol·K.
T = Temperature (In kelvin)
n = no. of moles.



Real gas ⇒ Ideal gas ⇒ Low Temp Pressure
High Temp

1 mole of any Gas: $PV = RT$

Ideal Gas $= \frac{PV}{RT} = 1$ (Low P & High T)

High temp ↑↑ real gas & ideal gas get closer

10^{-3} kg of O_2
Ideal gas
At all pressure fall Temp
$R = 8.14$ J/mol K
$\frac{PV}{T} = ?$
where .... at y axis.

$\frac{PV}{T} = ?$

$PV = nRT$

$\frac{PV}{T} = nR$

$n = \frac{mass}{molar mass} \times R$

$n = \frac{10^{-3} kg}{32 g} \times 8.314$

$= \frac{1 g}{32 g} \times 8.314$

$= 0.25$

For what mass of H_2 gas the value of $\frac{PV}{T}$ will be same as $\frac{PV}{T}$ for 10^{-3} kg of O_2 gas

→ $PV = n_{O_2} RT$

$PV = n_{H_2} RT$

$\left(\frac{PV}{T}\right)_{O_2} = \left(\frac{PV}{T}\right)_{H_2}$

$n_{O_2} R = n_{H_2} R$

$\frac{\text{mass of } O_2}{\text{molar mass } O_2} = \frac{\text{mass of } H_2}{\text{molar mass } H_2}$

$\frac{10^{-3} kg}{_{16}32 g} = \frac{x}{{}_1 2 g}$

$\frac{1}{16} \times 10^{-3} kg = x$

$x = \frac{100}{16} \times 10^{-5} kg$

$= 6.25 \times 10^{-5} kg$

$m_{H_2} = 6.25 \times 10^{-5} kg$

Q Find the ratio of $\frac{\text{molecular vol}^m}{\text{Actual vol}^m \text{ occupied}}$ by O_2 molecules.

(diameter of O_2 molecule is $3 A^\circ$.

→ 1 mole of O_2 at STP

$= 22.4 L$

$= 22.4 \times 10^{-3} m^3$

6.022×10^{23} molecules of O_2

$V = \frac{4}{3} \pi r^3 \times 6.022 \times 10^{23}$

$\therefore \frac{\text{Molecular vol}^m}{\text{Actual vol}^m \text{ occupied}} = \frac{4/3 \pi r^3 \times 6.022 \times 10^{23}}{22.4 \times 10^{-3} m^3}$

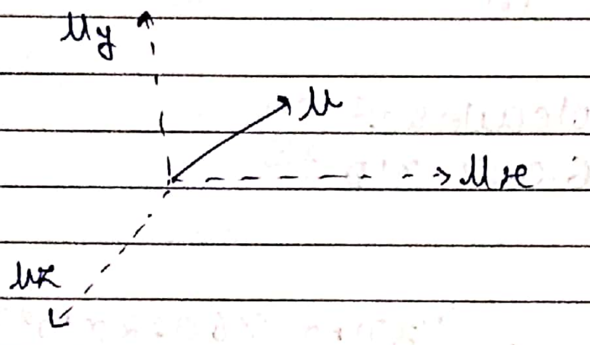
Kinetic Theory of Gases:

Postulates:

i] Gas is made up of atoms & molecules.

ii] Molecules of same gases are identical in all respects. (mass, shape, size)

iii] Molecules are constanly in random motion along straight line.



$$u = u_x\hat{i} + u_y\hat{j} + u_z\hat{k}$$

$$u_x = u_y = u_z$$

$$|u| = \sqrt{u_x^2 + u_y^2 + u_z^2}$$

$$u^2 = u_x^2 + u_y^2 + u_z^2 = 3u_x^2$$

iv] All the coll^n s of gas molecules among themselves & wall of container is elastic in nature

- Momentum
- Kinetic energy

} No loss.

v] The pressure of a gas is due to coll^n s of molecules with wall of container.

P α No. of coll^n s of molecules per unit area

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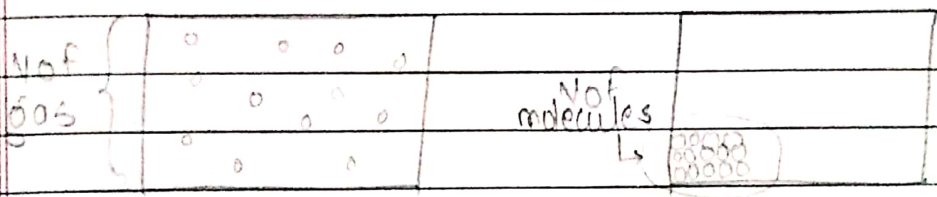
vi] The kinetic energy of gas depends only & only upon temp (absolute)
(doesn't depend on nature of gas)

vii] The vol^m occupied by gas molecules is negligible when compared to vol^m of gas.



viii] There is no inter molecular force of attraction attraction among molecules.

ix] Gravity is neglected.

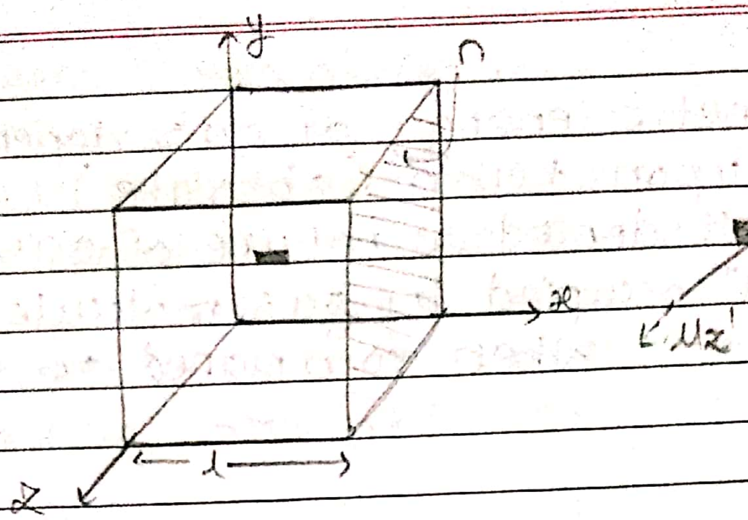
* Kinetic gas eq^n :

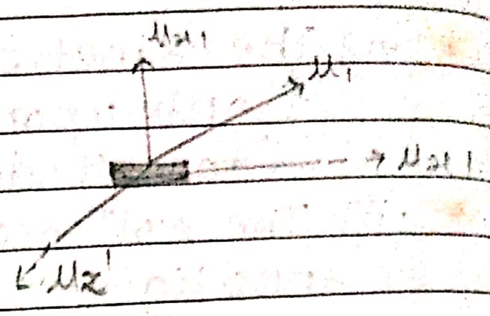
$$PV = \frac{1}{3} m n u_{rms}^2$$

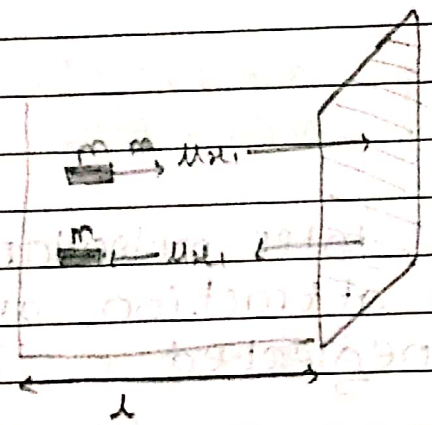
m → mass of one molecule of gas
n → no. of molecules
u_{rms} → Root mean square velocity of molecules

$$u_{rms} = \sqrt{\frac{u_1^2 + u_2^2 + u_3^2 + \ldots u_n^2}{n}}$$

K.E of n moles of gas $= n \times \frac{3}{2} RT$







$P_i = m u_{x1}$

$P_f = -m u_{x1}$

$\Delta P = 2 m u_{x1}$

time taken for each coll^n	$= \dfrac{2l}{u_{x1}}$

Force exerted on wall $= \dfrac{\Delta P}{\Delta t}$

$$\vec{F} = \frac{-2 m u_{x1}}{\frac{2l}{u_{x1}}}$$

$$\boxed{\vec{F} = \frac{m u_{x1}^2}{l}}$$

$* \; u = u_x \hat{i} + u_y \hat{j} + u_z \hat{k}$

$u^2 = 3u_x^2$

$$\vec{F} = \frac{m}{l} \frac{1}{3} u_1^2$$

$$\vec{F} \text{ due to one molecule} = \frac{m}{l} u_{x_1}^2 = \frac{m}{l} \frac{1}{3} u_1^2$$

F due to 'n' molecules:

$$= \frac{1}{3}\frac{m}{l}u_1^2 + \frac{1}{3}\frac{m}{l}u_2^2 \ldots + \frac{1}{3}\frac{m}{l}u_n^2$$

$$F_{net} = \frac{1}{3}\frac{m}{l}\left[\frac{u_1^2 + u_2^2 + \ldots\ldots u_n^2}{n}\right] \times n$$

$$F_{net} = \frac{1}{3}\frac{mn}{l} u_{rms}^2$$

$$\text{Pressure} = \frac{F}{A} = \frac{1}{3}\frac{mn\,u_{rms}^2}{l \times l^2} = \frac{1}{3}\frac{mn\,u_{rms}^2}{l^3}$$

$$P = \frac{1}{3}\frac{mn\,u_{rms}^2}{V}$$

Kinetic Gas eq^n

$$PV = \frac{1}{3} mn\,u_{rms}^2$$

$PV = nRT$
0.0821 atm L/mol k

$K.E = \frac{3}{2} RT$
8.314 J/mol k

Kinetic energy:

$K.E = \frac{1}{2} m u_1^2$
(of 1 molecule)

K.E of n molecules $= \frac{1}{2} m u_1^2 + \frac{1}{2} m u_2^2 + \dots + \frac{1}{2} m u_n^2$

$= \frac{1}{2} m \frac{(u_1^2 + u_2^2 + u_3^2 + \dots + u_n^2)}{n}$

K.E of n molecules $= \frac{1}{2} m u^2_{rms} \times n$

$\frac{K.E}{PV} = \frac{\frac{1}{2} m u^2_{rms} \times n}{\frac{1}{3} m n u^2_{rms}}$

$PV = \frac{2}{3} K.E$

$K.E = \frac{3}{2} PV$ for 1 mole of gas(ideal) [$PV = RT$]

$K.E = \frac{3}{2} RT$

$PV = \frac{1}{3} m n u^2_{rms}$ — m: mass of 1 molecule; n: no. of molecules; rms

K.E of 1 mole of ideal gas $= \frac{3}{2} RT$ — R: Gas constant 8.314 J/mol k

$\frac{R}{N_A}$ - Boltzmann constant

K.E of n moles of ideal gas	$= n \times \frac{3}{2} RT$

K.E of 1 molecule	$= \frac{3}{2} \frac{RT}{N_A} = \frac{3}{2} KT$

↑ Boltzman constant

$K = 1.38 \times 10^{-23}\ J/K$

Q.1 Find out K.E of 8g of CH_3 at 27°C.

→ $K.E = n \times \frac{3}{2} RT$

$= 0.5 \times \frac{3}{2} \times 8.314 \times 300\ J$

$\text{no. of moles} = \frac{\text{mass}}{\text{molar mass}}$

$= \frac{8}{16} = 0.5$

Q.2] Find out K.E of 1 molecule of oxygen(O_2) gas at 127°C.

→ $K.E = \frac{3}{2} KT$

$= \frac{3}{2} \times 1.38 \times 10^{-23} \times 400$

$= 600 \times 1.38 \times 10^{-23}\ J$

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Q.3 Find the change in K.E of 1 mole of ideal gas when temp changes by 50°C

→ $\Delta K.E = KE_f - K.E_i$

$= \frac{3}{2}RT_2 - \frac{3}{2}RT_1$

Me

$= \frac{3R}{2}[\quad]$

Sir

$KE_i = \frac{3}{2}RT$

$K.E_f = \frac{3}{2}R(T+50)$

$\Delta KE = KE_f - K.E_i$

$= \frac{3}{2}R(T+50-T)$

$= \frac{3}{2}R \times 50$

$= \frac{3}{2} \times 8.314 \times 50$ J

Q4 At what temp, the KE will be half of its value at ~~120°C~~ 127°C.

→ $K.E_{127°C} = n \times \frac{3}{2} \times R \times 400$

$K.E_T = \frac{K.E_{127°C}}{2}$

$n \times \frac{3}{2} \times R \times T = \frac{n \times 3/2 \times R \times 400}{2}$

$T = 200K$

$1 amu = 1.67 \times 10^{-27} kg$

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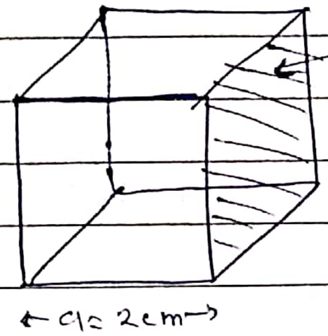
Short cut
KE ∝ T

Q5



They make 500 coll^{n}s with the wall in each sec.

Find the temp.

→ Time for 1 coll^n = $\frac{1}{500}$ s

time taken for 1 coll^n = $\frac{2a}{u_{rms}}$

$$\frac{1}{500} = \frac{2a}{u_{rms}}$$

$$\frac{1}{500} = \frac{2 \times 2 \times 10^{-2}}{u_{rms}}$$

$$u_{rms} = 20 m/s$$

K.E of 1 molecule $= \frac{1}{2} m u_{rms}^2$

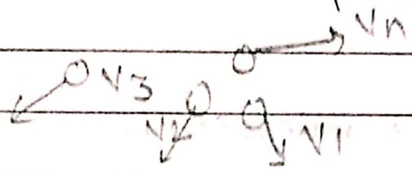
$$= \frac{3}{2} kT$$

$$\frac{1}{2} m u_{rms}^2 = \frac{3}{2} k T$$

[1 molecule of He = 4 amu
$= 4 \times 1.67 \times 10^{-27}$ kg]

$M = m N_A$
(molar mass) (1 molecule mass)

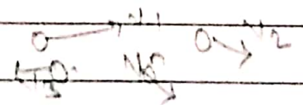
RMS Velocity
[Root Mean Square]



$$V_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + \dots + v_n^2}{n}}$$

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$
(m/s)

$R = 8.314$ J/mol K
T = in Kelvin
M = Mass i.e molar mass of gas (in kg)



$$K.E_{Total} = \frac{1}{2} m v_1^2 + \frac{1}{2} m v_2^2 + \frac{1}{2} \dots + \frac{1}{2} m v_n^2$$

$$= \frac{1}{2} m \left(\frac{v_1^2 + v_2^2 + \dots v_n^2}{n} \right) \times n$$

$$= \frac{1}{2} m V_{rms}^2 \times n$$

K.E of 1 mole of gas $= \frac{1}{2} m v_{rms}^2 \times N_A$

$$\frac{3}{2} RT = \frac{1}{2} M v_{rms}^2$$

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$
(in kg)

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Q. V_{RMS} For H_2 molecules at $27°C$.

→ $V_{RMS} = \sqrt{\frac{3RT}{M}}$

$= \sqrt{\frac{3 \times 8.314 \times 300}{2 \times 10^{-3}}}$

$= \sqrt{3 \times 4 \times 300 \times 10^{3}}$

$= \sqrt{36 \times 10^{5}}$

$= \sqrt{3.6 \times 10^{6}}$

$= 1.3 \times 10^{3}$

Q if V_{rms} for H_2 is x at a given Temp. Find V_{rms} for O_2 at given Temp.

→

$\frac{V_{O_2}}{V_{H_2}} = \frac{\sqrt{\frac{3RT}{M_{O_2}}}}{\sqrt{\frac{3RT}{M_{H_2}}}} = \sqrt{\frac{M_{H_2}}{M_{O_2}}}$

$\frac{V_{O_2}}{x} = \sqrt{\frac{2 \times 10^{-3}}{32 \times 10^{-3}}} = \sqrt{\frac{1}{16}}$

$V_{O_2} = \frac{x}{4}$

$V_{RMS} \propto \sqrt{T}$

Q At 300°K, V_{rms} of H_2 is same as V_{rms} of O_2 at T (in kelvin). Find T.

→ $V_{rms\,H_2} = V_{rms\,O_2}$

$$\sqrt{\frac{3RT}{M_{H_2}}} = \sqrt{\frac{3RT}{M_{O_2}}}$$

$$\sqrt{\frac{300}{2 \times 10^{-3}}} = \sqrt{\frac{T}{32 \times 10^{-3}}}$$

$$\sqrt{300} = \sqrt{\frac{T}{16}}$$

$$300 = \frac{T}{16}$$

$$T = 4800K$$

Pressure

$$V_{RMS} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3P}{\rho}} \text{ (S.I unit)}$$

Derivation:

K.E of 1 mole of Gas $= \frac{1}{2} M V_{rms}^2$

$$\frac{3}{2} RT = \frac{1}{2} M V_{rms}^2$$

$$\frac{3}{2} PV = \frac{1}{2} M V_{rms}^2$$

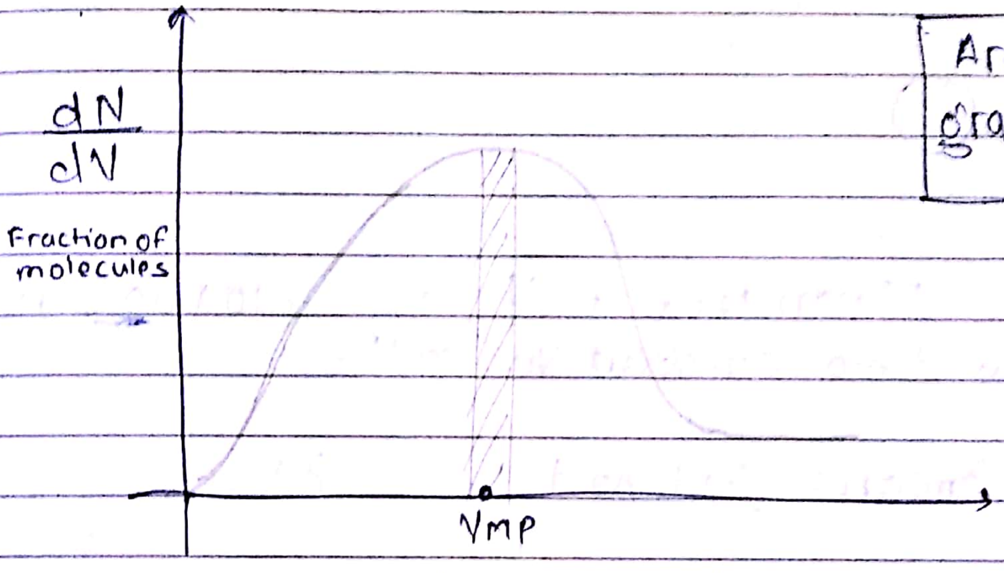
$$V_{rms} = \sqrt{\frac{3PV}{M}} = \sqrt{\frac{3P}{(M/V)}}$$

$$= \sqrt{\frac{3P}{\rho}}$$

At constant temp

$P \uparrow \quad V_{rms}$ = constant.

Maxwell's distribution of velocities



Area of graph = Total no. of molecules

$$V_{MP} = \sqrt{\frac{2RT}{M}}$$

$$V_{RMS} = \sqrt{\frac{3RT}{M}}$$

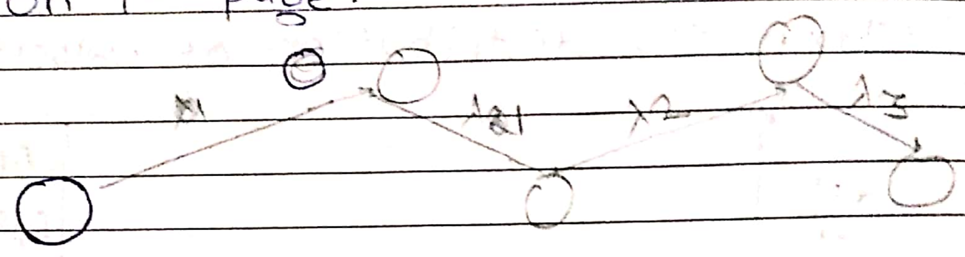
(capital) $$V_{avg} = \sqrt{\frac{8RT}{\pi M}}$$

(velocity) $$\vec{V}_{avg} = 0$$

R > A > M

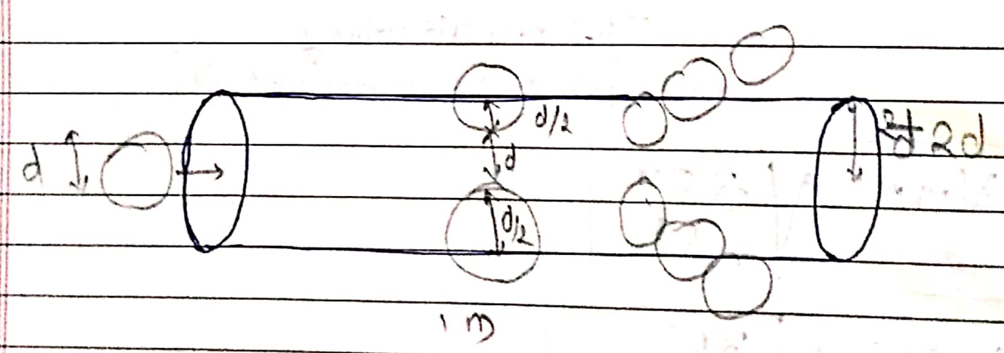
$V_{RMS} > V_{avg} > V_{MP}$

Mean Free Path:
On 1st page.



Mean free path is average distance b/w two consecutive coll^{n}s.

$$\lambda_{mean} = \frac{\lambda_1 + \lambda_2 + \dots \lambda_n}{n}$$



no. of molecules per unit vol^m = x

$1m^3 = x$
$2m^3 = 2x$
$V m^3 = Vx$

no of molecules inside cylinder = $\pi(d)^2 \times 1 \times x$

no of coll^{n}s in 1m travelling distance = $\pi(d)^2 x$

$\pi d^2 x$ coll^{n}s $\longrightarrow$ 1 m distance
1 coll^n $\longrightarrow$ $\frac{1}{\pi d^2 x}$ distance.

$$\lambda = \frac{1}{\pi d^2 x}$$

$$\lambda = \frac{RT}{\pi d^2 P N_A}$$

N_A → Avogadro's no.
P → pressure in SI unit
d → diameter of 1 molecule

x → no of molecules per unit vol^m

$$PV = nRT$$

$$\frac{n}{V} = \frac{P}{RT}$$

$$x = \frac{n}{V} \times N_A$$

$$x = \frac{P N_A}{RT}$$